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ON THE QUANTITATIVE SCALAR OR-IMPLICATURE

ABSTRACT. Two simple generalized conversational implicatures are investigated: (1) the quantitative scalar implicature associated with 'or', and (2) the 'not-and'-implicature, which is the dual to (1). It is argued that it is more fruitful to consider these implicatures as rules of interpretation and to model them in an algebraic fashion than to consider them as nonmonotonic rules of inference and to model them in a proof-theoretic way.

1. AN IMPLICATURE CONCERNING 'OR'

The aim of this paper is to formalize one of the Gricean *Generalized Conversational Implicatures* (*GCI*'s). This is a particular instance of a larger program of attempting to 'formalize pragmatics'.

Levinson in his book cites certain general distinctive properties of *GCI*s. The following of these are of particular relevance for our purposes (Levinson 2000, 15):

1. Cancellability.
2. Nondetachability.
3. Calculability.

The first of these is connected with the nonmonotonic aspects of *GCI*s. The second property means that "any expression with the same coded content will tend to carry the same implicatures".[1] It implies that we can restrict our attention to what is linguistically given and do not have to look at aspects of the nonlinguistic context in which the relevant assertions are made. The third property means that natural language users must possess something like an algorithm that allows them to determine in a given (knowledge) context which implicatures he/she may legitimately (albeit defeasibly) draw.

A problem with *GCI*s is that many of them are very open-ended (Levinson 2000, 49) and that it is not always clear how they ought to be applied in particular contexts. But perhaps a particularly clean and simple test-case would be the following *GCI* associated with the logical connective 'or' (Levinson 2000, 18):[2]

If a speaker says $A \vee B$, then it is *implicated* that $\neg(A \wedge B)$.

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Let us from now on pretend that this is the only implicature associated with 'or', and call it *the* $\vee$-implicature. This implicature is an instance of the general species of *GCI*s associated with scalar expressions (Levinson 2002, 35–37). For instance, there is an analogous implicature concerning $\exists$:

> If a speaker says $\exists x A(x)$, then it is implicated that $\neg\forall x A(x)$.

An advantage of the implicature concerning 'or' is that it can be investigated within a logically particularly simple context, i.e., the context of propositional logic.

The aim of *GCI*s in general, and of the $\vee$-implicature in particular, is that they increase the informational content of what is communicated.[3] It is thought that in general, information production (primarily by speech) is much slower than speech comprehension. *GCI*s attempt to narrow this discrepancy by increasing the communicative content of what is said.

There is a discussion in the literature about whether these implicatures contain an *epistemic* component, and, if so, how this epistemic component should be expressed (Levinson 2000, 75–79).[4] Defenders of the epistemic reading of the $\vee$-implicature would take issue with the way it was expressed above. They would express the implicature as

> If a speaker says $A \vee B$, then it is implicated that *the speaker believes* $\neg(A \wedge B)$.

or perhaps as:

> If a speaker says $A \vee B$, then it is implicated that *the speaker does not believe* $A \wedge B$.

In this paper, we opt for a *nonepistemic* reading of the $\vee$-implicature, and therefore adhere to the original formulation. We take it that when a speaker asserts $A \vee B$, he implicates that $A \wedge B$ is not the case in the world. This implicature then triggers a *second* implicature, namely that the speaker believes the first implicature, but that is another matter. It falls outside the scope of the present paper to defend the non-epistemic position at length.[5] Instead, we want to see where it takes us and how it can best be made mathematically precise.

We will start by making various simplifying assumptions. Some of these are later removed; others are left in place throughout the paper. Hopefully this will eventually lead us to a fairly satisfactory mathematical treatment of the implicature. But we are not under the illusion that this paper is more than a first step in this direction. The upshot of this paper is that the logical modeling of even the simplest of the *GCI*s is a complicated task.

2. THE IMPLICATURE AS A NONMONOTONIC RULE

Levinson suggests that *GCI*s should be modelled as nonmonotonic inference rules (Levinson 1970, 45–49). Let us try to see how this would go.

We work in the Fitch-style natural deduction formulation *CPL* of classical propositional logic.[6] $A \rightarrow B$ is to be read simply as an abbreviation of $\neg A \vee B$. In this paper we will not address the difficult question of the proper treatment of the conditional expressions in natural language and the interplay between conditionals and disjunction.[7]

A first proposal would be to express the $\vee$-implicature as a default rule in the context of Reiter's *default logic*.[8] This would amount to adding to *CPL* the following default rule:

$$(A \vee B : \neg (A \wedge B) / \neg (A \wedge B)).$$

In words: if one has found that $A \vee B$, then if it is consistent with what has been obtained that $\neg (A \wedge B)$, one may conclude to $\neg (A \wedge B)$. One can straightforwardly reformulate this rule as a natural deduction rule with a consistency condition.

But there is a problem with this proposal. The default rule should be restricted to *asserted* sentences. We need to avoid that for every sentence of the form $A \vee B$, language users are allowed to conclude $\neg (A \wedge B)$, for this is not intended by the Gricean implicature concerning 'or'. For instance, when a sentence $A \vee B$ is hypothetically uttered, the implicature is not in force.[9] Therefore we must ensure that in hypothetical subproofs the default rule cannot be applied.

The following rule of inference, which we call $C\vee$, takes this caveat into account. We formulate it straightaway in natural deduction format:

A_1 Premise
A_2 Premise
$\vdots$
n. A_n Premise
$\vdots$
k. $A \vee B$
k + 1. $\neg (A \wedge B)$ 1-k, $C\vee$.

The following two restrictions apply to applications of $C\vee$:

1. $C\vee$ may only be applied to formulas in the *main proof*, not to hypothetical statements, i.e. formulas in subproofs.
2. Let $l_1, \ldots, l_m$ be an enumeration of all formulas in the main proof from line 1 to m. Then $C\vee$ may only be applied to $A \vee B$ in case

$$l_1, \ldots, l_m \nvdash_{CPL} A \wedge B.$$

Condition 1 restricts $C\vee$ to *asserted* sentences. Condition 2 embodies the defeasible or nonmonotonic character of the implicature $C\vee$. One is only permitted to apply the implicature if the result does not conflict with previously obtained information. It involves a *consistency check*. In the propositional case, this condition is effectively computable, whereby Levinson's calculability requirement on *GCI*s is satisfied.[10] But for the corresponding consistency check for the analogous *GCI* for $\exists$, in a (polyadic) quantificational setting, undecidability would creep in. For this reason, Batens and Meheus in their Adaptive Logics[11] 'unwind' such consistency checks: roughly, one is only prevented from applying an adaptive rule to infer A if $\neg A$ has not *actually* been derived earlier in the proof. This makes their adaptive inference rules computationally tractable. Their logics allow agents to reason in an (implicitly) inconsistent way for a while, until the inconsistency comes to the surface. The proofs of our system $CPL + C\vee$ then roughly correspond to the subclass of *consistent* adaptive proofs.

It must be conceded that Batens' and Meheus' adaptive systems more accurately reflect the way in which people *actually* reason. The strong consistency check incorporated in Condition 2 on the application of $C\vee$ must therefore be seen as a simplifying idealization.

3. ELEMENTARY PROPERTIES OF THE RULE

The first thing we can note is that the rule $C\vee$ gives rise to some completely trivial, but peculiar *dualities*. First, we have:

$$\frac{A \vee \neg A}{\neg (A \wedge \neg A)}$$

by an *immediate* application of $C\vee$. Second, if we suppose for a moment that $\rightarrow$ is a primitive connective of our language governed by the usual classical natural deduction rules, we have:

$$\frac{A \rightarrow B}{B \rightarrow A}.$$

These inferences are again allowed only in the main proof, and under the condition that the inference does not introduce an inconsistency.

Let us concentrate for a moment on the second of the above default rules: call it $C \rightarrow$.[12] In fact, $C\vee$ and $C \rightarrow$ are *equivalent implicatures*:

PROPOSITION 1. $C\vee \Longleftrightarrow C \rightarrow$.

And indeed, in linguistic practice conditional statements *are* often read as biconditionals.

According to what can be called the *Gricean Master Scheme* (*GMS*), all pre-theoretical intuitions concerning correctness of reasoning with logical expressions must be explained by means of a combination of:

Classical Logic (*Part I*) + Conversational Implicatures (*Part II*).

In particular, according to *GMS* the so-called *paradoxes of the material implication* should be explained in terms of breach of implicatures.

GMS of course goes back to the work of Grice.[13] But variants of it have been defended by several influential linguists, philosophers of language and pychologists. For example, David Lewis for a long time thought that the meaning of conditionals in natural parlance is given by the material implication.[14] Johnson-Laird and Byrne also defend a variant of this position.[15]

An alternative explanatory strategy just postulates that there are several different meanings of expressions like 'or' and 'if ... then' in natural language, and that it depends on the particular context of use which one is intended. One may wonder how one could ever empirically decide between these two different general approaches.

But note that in the light of the Proposition 1, *GMS* comes one step closer to being empirically testable. The idea is this. It is empirically observable in language use whether and *how strong* an implicature holds. In this sense, $C\vee$ and $C \rightarrow$ are independently testable. But since $A \rightarrow B$ is in virtue of Part I of *GMS* equivalent to $\neg A \vee B$, there is a *logical* connection between $C\vee$ and $C \rightarrow$. In other words, from the empirical fact $C\vee$, $C \rightarrow$ is *predicted*. Specifically, in a context where the implicature $C\vee$ is justified for a formula $A \vee B$, one would expect that the implicature $C \rightarrow$ can with equal justification be applied to $\neg A \rightarrow B$. I hasten to add to this that I am not under the illusion that it would be a simple matter to implement such an empirical test. Also, I am aware that few neo-Griceans would today be willing to defend *GMS* in the naive form in which it was presented here.[16]

Let us understand the notion of a *proof* in $CPL + C\vee$ in the obvious way. Then we can define a *weak* notion of consequence:

DEFINITION 2. $A_1, \ldots, A_n \vdash^{w}_{C\vee} A \equiv$ *there is a proof in* $CPL + C\vee$ of A from $A_1, \ldots, A_n$.

Let us call a proof consistent if no contradiction can be derived from it (considered as a set of sentences) by means of CPL alone.

PROPOSITION 3. If $\{A_1, \ldots, A_n\}$ is consistent, then every $CPL + C\vee$-proof from $A_1, \ldots, A_n$ is consistent.

Proof. We only need to show that $C\vee$ is consistency-preserving. But this follows immediately from condition 2 on $C\vee$. □

Note that we do not have, in general, that if $\mathcal{B}_2 \circ \mathcal{B}_1$ are proofs in CPL+$C\vee$ (from the same premises), then $\mathcal{B}_2 \circ \mathcal{B}_1$ is a proof in $CPL + C\vee$. For $\mathcal{B}_1$ may contain conditions blocking some of the $\mathcal{B}_2$-inferences. As a simple example, suppose $\mathcal{B}_2$ is:

1. A Premise
2. $A \vee B$ Premise
3. $A \vee \neg B$ Premise
4. $\neg (A \wedge B)$ 1–2, $C\vee$

And let $\mathcal{B}_2$ be:

1. A Premise
2. $A \vee B$ Premise
3. $A \vee \neg B$ Premise
4. $\neg (A \wedge \neg B)$ 1–3, $C\vee$

Then $\mathcal{B}_2 \circ \mathcal{B}_1$ is not a $CPL + C\vee$-proof: its second application of $C\vee$ violates the associated consistency requirement. This phenomenon illustrates a nonmonotonic feature of the or-implicature.

From now on, when we speak of proofs, we mean $CPL + C\vee$-proofs. We can formulate a *strong* notion of consequence.

DEFINITION 4. $A_1, \ldots, A_n \vdash^{s}_{C\vee} A \equiv$ for every proof $\mathcal{B}$ from $A_1, \ldots, A_n$, there is a proof $\mathcal{B}' \supseteq \mathcal{B}$ which proves A.

This allows us to define the strong consequence set $\mathcal{SC}(A_1, \ldots, A_n)$ of a set of premises $A_1, \ldots A_n$:

DEFINITION 5. $\mathcal{SC}\,(A_1, \ldots, A_n) \equiv \{A \mid A_1, \ldots, A_n \vdash^{s}_{C\vee} A\}$.

PROPOSITION 6. (Conservativeness of $C\vee$ over classical propositional logic) $\mathcal{SC}\,(A_1, \ldots, A_n) = CPL + A_1, \ldots, A_n$.

Proof. By a reductio ad absurdum. Suppose $A_1, \ldots, A_n \nvdash_{CPL} B$. Let $\top$ be an arbitrary propositional tautology. And let $\mathscr{B}$ be the following proof:

A_1
$\vdots$
n. A_n
n + 1. $\top$ tautology
n + 2. $\top \vee B$ $\vee$-Introduction
n + 3. $\neg(\top \wedge B)$ $C\vee, 1 - n + 2$
n + 4. $\neg B$

The step from n+2 to n+3 is justified, since $A_1, \ldots A_n \nvdash_{CPL} \top \wedge B$, for otherwise $A_1, \ldots A_n \vdash_{CPL} B$, contradicting our assumption. Now no proof $\mathscr{B}'$ proves B: otherwise $\mathscr{B}'$ would be an inconsistent proof. By the previous proposition its set $A_1, \ldots, A_n$ of premises would have to be inconsistent. But by the supposition of the present proof they must be consistent. So we are done. □

This shows that as a rule, $C\vee$ is in the context of classical propositional logic *essentially* defeasible: there can always be background information in the context of which it cannot be applied.

The argument for our last proposition indicates that there is something *wrong* with our treatment of the implicature as a nonmonotonic rule. For it shows how given a list of premises $A_1, \ldots, A_n$, *any* B which is consistent with $A_1, \ldots, A_n$ can be derived from it using $C\vee$. This accords ill with the way in which the implicature is used in ordinary communication.

The argument for our last proposition highlights the crucial role of the propositional rule of $\vee$-*Introduction* in the present setting. It shows that problematic applications of $C\vee$ arise when a language user *first* weakens the semantic information content that is given to him, and *subsequently* applies CV to obtain highly informative propositions.

One can of course try to eliminate the undesirable consequences by imposing further restrictions on applications of $C\vee$. But this would make the nonmonotonic rule rather complicated, compared to the apparent simplicity of the rule as it is used in daily communication. So let us try to model the implicature in another way.

4. AN ALGEBRAIC APPROACH

In this section, we try to model the $\vee$-implicature as an operator which acts on a simple algebraic structure. We will try to show that an algebraic treatment allows us to capture more of the subtle aspects of the implicature.

Our algebraic structure is a *partial ordering* $(F, \leq)$. The elements of F are the propositional formulas in the alphabet $p_1, \ldots, p_n, \ldots$. The relation $\leq$ orders the elements of F by *information content*:

$$A \leq B \equiv \models B \rightarrow A.$$

With each $A \in F$, we can then associate an *information level* $i\,[A]$ of A:

DEFINITION 7. $i\,[A]$ is the equivalence class of all logical equivalents of A in the alphabet $p_1, \ldots, p_n, \ldots$. And we say that $i\,[A] \leq i\,[A]$ iff $A \leq B$.

The idea now is that the $\vee$-implicature acts as an operator $\mathcal{O}$ on $(F, \leq)$, such that *typically*, for an $A \in F$ which is of disjunctive form, the implicated result $A' = A \wedge \mathcal{O}\,(A)$ is informationally stronger than A itself:

$$i\,[A] < i\left[A'\right].$$

This would mean that the $\vee$-implicature indeed increases informational content.

A couple of remarks are in order here. First, note that in our partial ordering (p.o.), $\leq$ is the *inverse* of the usual ordering of propositions in the atomic boolean algebra generated by $p_1, \ldots, p_n$. In the p.o. $(F, \leq)$, $\top$ (*verum*) is a bottom element and $\perp$ (*falsum*) is a top element. Second, we do *not* follow the usual practice of taking equivalence classes (induced by logical equivalence) as elements of our p.o. The reason for this is that the $\vee$-implicature of two logically equivalent formulas can be quite different:

EXAMPLE 8. Trivially,

$$\models p \vee q \leftrightarrow (p \wedge \neg q) \vee q.$$

When the $\vee$-implicature is applied to $p \vee q$ we obtain $\neg\,(p \wedge q)$; when the implicature is applied to $(p \wedge \neg q) \vee q$ we obtain $\neg\,((p \wedge \neg q) \wedge q)$, i.e. $\top$. But evidently:

$$\not\models ((p \vee q) \wedge \neg\,(p \wedge q)) \leftrightarrow \left(\left[(p \wedge \neg q) \vee q\right] \wedge \top\right).$$

DEFINITION 9. The *communicated content* of an expression A is defined as the semantic content of A *plus* the implicature(s) of A.[17]

In other words, the communicated content of $p \vee q$ differs from that of $(p \wedge \neg q) \vee q$.

The previous example then shows in effect that there are formulas A, B which are logically equivalent but which have respective communicative contents A', B' such that $i\left[A'\right] < i\left[B'\right]$.

The following example shows that even difference in the position of brackets in a purely disjunctive formula can yield different immediate $\vee$-implicatures:

EXAMPLE 10. Suppose $D = p_1 \vee p_2 \vee p_3$. If we formulate D as $(p_1 \vee p_2) \vee p_3$, then we can apply the implicature to obtain $\neg\left[(p_1 \vee p_2) \wedge p_3\right]$. If we formulate D as $p_1 \vee (p_2 \vee p_3)$, then the implicature yields $\neg\left[p_1 \wedge (p_2 \vee p_3)\right]$. If we formulate D as $(p_1 \vee p_3) \vee p_2$, then the implicature yields $\neg\left[(p_1 \vee p_3) \wedge p_2\right]$. These three implicatures are nonequivalent.

This last example illustrates a more general phenomenon. There is a tendency to say that in common discourse, *all* these applications of the implicature are admissible when a speaker assertively utters "$p_1 \vee p_2 \vee p_3$", i.e. *both* disjuncts tend to be read exclusively. This gives us a compact way of conveying information which otherwise would take a long sentence to express in terms of negation and (inclusive) disjunction. Nevertheless, the empirical evidence for this construal of the $\vee$-implicature is admittedly limited – we hardly ever reason with disjunctions of more than three disjuncts. But if we accept this, then we want to define $\mathcal{O}(D)$ as the conjunction of D with applications of the implicature to *all* of these equivalent ways of formulating D. Of course, this is again an idealization: actual language users apply one implicature at a time. Eventually one will want a more fine-grained approach, which unwinds $\mathcal{O}$ in its individual applications of the 'or'-implicature.

To define $\mathcal{O}$ formally involves some terminology but is basically straightforward.

DEFINITION 11. Let $\mathcal{F}_n$ be the collection of all the $(2^n - 2)$ functions $f : \{1, \ldots, n\} \mapsto \{0, 1\}$ without the (for our purposes degenerate) constant functions.

The functions $f \in \mathcal{F}_n$ correspond to the different ways in which a disjunctive formula D consisting of n disjuncts can be formulated:

Suppose D is a disjunctive formula. Then for any $f \in \mathcal{F}_n$, let

$$D_f \equiv \left(D_{i_1} \vee \cdots \vee D_{i_k}\right) \vee \left(D_{j_1} \vee \cdots \vee D_{j_l}\right),$$

with $f(i_1) = \cdots = f(i_k) = 1$ and $f(j_1) = \cdots = f(j_l) = 0$.

Now we can define the local action of the implicature on a *particular* formulation of D:

DEFINITION 13. Again suppose D is a disjunctive formula. For any $f \in \mathcal{F}_n$, let

$$\mathcal{O}^p\left(D_f\right) \equiv \neg\left[\left(D_{i_1} \vee \cdots \vee D_{i_k}\right) \wedge \left(D_{j_1} \vee \cdots \vee D_{j_l}\right)\right]$$

In terms of these particular implicatures, we can finally define the *global* effect of the implicature on a disjunctive formula D:

DEFINITION 14. $\mathcal{O}(D) \equiv \bigwedge_{f \in \mathcal{F}_n} \mathcal{O}^p\left(D_f\right)$.

$\mathcal{O}(D)$ is then our proposed algebraic interpretation of the $\vee$*-implicature* of D. Then the communicative content $\mathcal{C}(A)$ of an assertion A can provisionally be expressed in terms of the operator $\mathcal{O}$. Let $A\left[g(D)/D\right]$ be the result of uniformly substituting *every* disjunctive subformula D of A (which is not a subformula of a larger disjunctive subformula of A) by $g(D)$. Then a provisional definition of the communicative content of A is:

DEFINITION 15. $\mathcal{C}^{\vee}(A) \equiv A\left[D \wedge \mathcal{O}(D)/D\right]$.

In contrast to our way of proceeding in Section 2, we do *not* here build into the definition that the $\vee$-implicature is not drawn (or drawn but retracted) from an asserted sentence A if $A\left[D \wedge \mathcal{O}(D)/D\right]$ is inconsistent outright or inconsistent with background knowledge of the hearer. For it seems to us that this is not part of the task of the algebraic model of the $\vee$-implicature *per se* but falls within the provinces of the discipline *belief revision*. We will see that nevertheless, the algebraic behavior of the $\vee$-implicature still features non-monotonic behavior.

Let us briefly look at some properties of $\mathcal{C}$.

As a first elementary observation, we have the following fact:

PROPOSITION 16 (idempotence of $\mathcal{C}^{\vee}$). For every A, $\mathcal{C}^{\vee}(A) = \mathcal{C}^{\vee}\left(\mathcal{C}^{\vee}(A)\right)$.

Proof. $\mathcal{C}(A)$ does not have the form of a disjunction. □

This means that the $\vee$-implicature is in a sense a non-iterative operation.

In connection with this, observe that the following form of iterated application of the $\vee$-implicature is excluded by our provisional definition:

$$p \vee \neg(q \vee r) \overset{\mathcal{O}}{\Rightarrow}$$

$$\neg(p \wedge \neg(q \vee r)) \overset{?\mathcal{O}}{\Rightarrow}$$

$$\neg(p \wedge \neg\neg(q \wedge r))$$

$$\Leftrightarrow \neg(p \wedge q \wedge r).$$

One might be inclined to allow the second application of the $\vee$-implicature, since in $\neg(p \wedge \neg(q \vee r))$ the subformula $q \vee r$ actually occurs positively. Nevertheless, it is not completely clear whether our intuitions sanction such iterations of the implicature. It seems that sentences with the required logical complexity for this question to arise are hardly ever asserted in natural language. So we will leave this question open in this paper.

Next, we have:

PROPOSITION 17. $\mathcal{C}^{\vee}$ is an inclusive, *nonmonotonic* operator on $(F, \leq)$.

Proof. That $\mathcal{C}^{\vee}$ is inclusive is immediate since $\mathcal{C}^{\vee}(A) \vDash A$. To see that $\mathcal{C}^{\vee}$ is not monotone, let p, q be two proposition letters. Then $(p \vee q) \leq q$, but

$$\mathcal{C}^{\vee}(p \vee q) = (p \wedge \neg q) \vee (\neg p \wedge q) \nleq \mathcal{C}^{\vee}(q) = q. \qquad \square$$

COROLLARY 18. For every D, $i[D] \leq i\left[\mathcal{C}^{\vee}(D)\right]$.

Proof. By the inclusiveness of $\mathcal{C}^{\vee}$. $\square$

Now let D be a formula. Then the content of D can in a familiar way be represented by means of a Venn diagram: the points in the Venn diagram represent the 'worlds in which D is true'. If D has the form of a disjunction consisting of n disjuncts, then the content of D is represented as the union of the Venn diagram-representations of the disjuncts. A moment's reflection shows that $\bigwedge_{f \in \mathcal{F}_n} \mathcal{O}^p\left(D_f\right)$ is then represented as the complement of the overlapping regions of the representations of the disjuncts D_i of D. In other words, $\mathcal{C}^{\vee}(D)$ is the union of the non-overlapping regions of the representations of the disjuncts of D. The present algebraic proposal takes the implicature to operate on the algebra of sentences, and *not* on the algebra of propositions expressed. Therefore the operator $\mathcal{C}^{\vee}$ divides the set of sentences into equivalence classes smaller than the relation of logical equivalence does. The question then arises whether this partition can somehow be informatively characterized. This question, which was raised by Hannes Leitgeb, is left open in this paper.

But there is the following complication. Given any asserted formula A, the $\vee$-implicature affects all and only the *positively* occurring disjunctive subformulas of A.[18] For instance, when a speaker asserts a sentence of the form

$$p \wedge (q \vee r),$$

or of the form

$$p \rightarrow (q \vee r),$$

then we also tend to interpret the occurrence of '$\vee$' in this formula exclusively. The fact that the or-implicature only applies to positively occurring disjunctive subformulas is emphasized in Noveck and Chierchia (2002).[19] In that article it is also emphasized that the or-implicature does not apply to *interrogative* sentences. And that, in our opinion, is in turn related to the fact that the implicature does not apply to *hypothetically asserted* sentences: hypothetical assertion-context can in a sense be regarded as *negative* contexts.

This suggests a revision of our definition of $\mathcal{C}^{\vee}$. Let us indicate a positive occurrence of a disjunctive formula D in A as D^{+}. Then the suggested revised definition is:

DEFINITION 19. $\mathcal{C}^{\vee}(A) \equiv A\left[D \wedge \mathcal{O}(D)/D^{+}\right]$.

5. THE $\vee$-IMPLICATURE AND THE $\overline{\wedge}$-IMPLICATURE

Neo-Gricean theories predict that if $\langle x_1, \ldots, x_n \rangle$ is a Horn scale on which a quantitative scalar implicature is based, then so is the dual $\langle \overline{x_n}, \ldots, \overline{x_1} \rangle$ of the scale (where $\overline{x}$ is the complement of x).[20] Now the $\vee$-implicature is based on the scale $\langle \vee, \wedge \rangle$. Therefore neo-Gricean theories predict a *dual* do the $\vee$-implicature, based on the scale $\left\langle \overline{\wedge}, \overline{\vee} \right\rangle$. This $\overline{\wedge}$-*implicature*, as we will call it, can roughly and informally be expressed as follows:

If a speaker asserts $\neg(A \wedge B)$, then it is *implicated* that $A \vee B$.

Here is a simple proposal for the way in which the $\overline{\wedge}$-implicature can be modeled.

Let C be a conjunctive formula $C_1 \wedge \cdots \wedge C_n$ with $n > 2$. Then we define:

DEFINITION 20. $\mathcal{A}(\neg C) \equiv C_1 \vee \cdots \vee C_n$.

DEFINITION 21. $\mathcal{C}^{\overline{\wedge}}(A) \equiv A\left[\neg C \wedge \mathcal{A}(\neg C)/(\neg C)^{+}\right]$.

Note that the definition of $\mathcal{A}(\neg C)$ is a bit simpler than the definition of $\mathcal{O}(D)$. The reason is that the way in which the brackets are placed in C do not matter here.

PROPOSITION 22. In general, we have:

a. $\not\models \mathcal{O}(\mathcal{A}(A)) \leftrightarrow A$.
b. $\models \mathcal{A}(\mathcal{O}(A)) \leftrightarrow A$

Proof. The proposition generalizes from the following argument:

$$\neg(p_1 \wedge p_2 \wedge p_3) \overset{\mathcal{A}}{\Rightarrow}$$

$$p_1 \vee p_2 \vee p_3 \overset{\mathcal{O}}{\Rightarrow}$$

$$\left\{\begin{array}{l} \neg(p_1 \wedge (p_2 \vee p_3)) \\ \neg(p_2 \wedge (p_1 \vee p_3)) \\ \neg(p_3 \wedge (p_1 \vee p_2)) \end{array}\right\} \overset{\mathcal{A}}{\Rightarrow}$$

$$\left\{\begin{array}{l} p_1 \vee p_2 \vee p_3 \\ p_1 \vee p_2 \vee p_3 \\ p_1 \vee p_2 \vee p_3 \end{array}\right\}$$

$$\Leftrightarrow p_1 \vee p_2 \vee p_3. \qquad \square$$

This means that $\mathcal{A}$ is a kind of *left adjoint* of $\mathcal{O}$ (in the algebraic sense) but $\mathcal{O}$ is not a left adjoint of $\mathcal{A}$.

To conclude, here is a simple-minded proposal for how the $\vee$-implicature and the $\overline{\wedge}$-implicature act together:

DEFINITION 23. $\mathcal{C}(A) \equiv A\left[D \wedge \mathcal{O}(D)/D^+; \neg C \wedge \mathcal{A}(\neg C)/(\neg C)^+\right]$.

Of course interaction with other implicatures deserves to be investigated as well. But this task again is deferred to future research.

6. INFERENCE OR INTERPRETATION?

On the algebraic way of modeling the $\vee$-implicature, it appears as a defeasible *rule of interpretation* of asserted linguistic utterances rather than as a nonmonotonic *rule of inference*. One aim of the present paper was to cast some doubt on the fruitfulness of exclusively proof-theoretic modeling of this implicature.[21] Indeed, it seems that language users do not apply the implicature much in extended nonmonotonic arguments – categorical or hypothetical. Rather, they use them as a tool of semantical interpretation of information that is conveyed to them.

We have shown how an algebraic representation of the $\vee$-implicature does exhibit *synchronic* nonmonotonic features. When the language user subsequently *reasons* deductively on the basis of the interpreted information, the implicature no longer plays a role.

As said before, the *dynamical* nonmonotonic character of the $\vee$-implicature comes into play when *after* applying the implicature, new information is obtained which contradicts the information that the language user has come to accept on the basis of the implicature. In such situations, language users revise their conclusions by canceling (some of) the implicature(s) which they have applied. A full account of the implicature ought to encompass not only the interaction of the $\vee$-implicature with other implicatures, but also the interaction with background knowledge. It seems likely that to model such logical behavior, systems of nonmonotonic reasoning (such as default logics, systems of belief revision, logical programming) *will* prove useful. But the issues that come up here are subtle. The following example shows that there are assertions for which it is consistent to apply the $\vee$-implicature to *some* positive subformulas of A, but not to all:

EXAMPLE 24. Suppose a speaker asserts

$$\big[(p \vee q) \wedge (q \vee r)\big] \wedge \big[(p \wedge q) \vee (q \wedge r)\big].$$

We have $\mathcal{O}\,(p \vee q) = \neg\,(p \wedge q)$ and $\mathcal{O}\,(q \vee r) = \neg\,(q \wedge r)$. Their conjunction

$$\mathcal{O}\,(p \vee q) \wedge \mathcal{O}\,(q \vee r)$$

then contradicts the second conjunct of the assertion.

In cases such as this example, the rule $\mathcal{C}^{\vee}$ predicts that we suspend all applications of the $\vee$-implicature.

And here (as at some places before) one more simplification of our account comes to the fore. In natural language, logically complicated sentences such as

$$\big[(p \wedge q) \vee (q \wedge r)\big] \wedge \big[(p \vee q) \wedge (q \vee r)\big]$$

(which we considered in an earlier example) are hardly every asserted. They are usually broken up into several less complicated sentences. Instead of uttering this complicated sentence, the speaker might first assert

$(p \wedge q) \vee (q \wedge r)$, and then goes on to assert $(p \vee q)$ and $(q \vee r)$. So at the moment when the third assertion is made, the first and the second assertion belong to the background knowledge. In such a situation the speaker perhaps does not apply the $\vee$-implicature to the *third* assertion. However this may be, this shows that our rules $\mathcal{C}^{\vee}$ and $\mathcal{C}^{\overline{\wedge}}$ are still simplifications. We have left the 'diachronic' complications mostly aside here because of our restricted aim of describing how the $\vee$-implicature applies to single free-standing assertions – this already proved to be complicated enough.

It would be unwarranted to conclude from the account that was presented here that interpreting an assertion *temporally precedes* deducing information from it. Determining the communicative content of an assertion involves taking implicatures into account. And to know, e.g., whether a particular $\vee$-implicature is in force involves carrying out a consistency check, which is an inferential task. it may well be that in practice language users continually go back and forth between interpreting and deducing.[22]

So I conclude that even the simplest of the implicatures display a behavior that is from a logical point of view actually quite intricate and interesting, and which is not easily modeled in a satisfactory manner. It seems that in order to achieve such a satisfactory treatment, several approaches need to be combined. It is of central importance in this context to arrive at a suitable division of labor.

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NOTES

1 This distinguishes *GCI*s from the so-called *Particular Conversational Implicatures* (*PCI*s). The so-called Relevance Theory of Sperber and Wilson (1986) does not accept this distinction.

2 See also Levinson (2000, 45–49).

3 See Levinson (2000, 27–29).

4 This discussion to some extent mirrors the debate in the *AI* community between the proponents of auto-epistemic logic on the one hand, and the defenders of default and circumscription logics on the other hand.

[5] For a more extensive defense of the approach taken here, the reader is referred to Atlas (1990).

[6] See, e.g., Thomason (1970, Chap. 4.)

[7] Nevertheless, this would be an important subject for further research. We make some remarks about this issue in the following section and in the concluding section.

[8] See, e.g., Horty (2001, 343–350).

[9] Noveck and Chierchia note that also when $A \vee B$ is uttered as a question, the or-implicature does not apply. See Noveck and Chierchia (2002). We come back to this issue in Section 4.

[10] Nevertheless, it ought to be mentioned that carrying out consistency checks for propositional logic is an NP-complete and therefore in the general case computationally intractable task.

[11] See Batens (2001).

[12] Joke Meheus pointed this duality out to me.

[13] See Grice (1989, 77ff).

[14] See Lewis (1976, 303–304).

[15] See Johnson-Laird and Byrne (2002).

[16] For a typically cautious neo-Gricean attitude towards conditionals, see Levinson (2000, 204–210).

[17] The distinction between implicature and communicated content is emphasized in Levinson (2000, 120–121).

[18] For a definition of positive versus negative occurrences of subformulas in a formula, see, e.g., Buss (1998, 15).

[19] Cf. Section 2.

[20] See, e.g., Levinson (2000, 64).

[21] This is however by far the most popular approach to date. One finds the proof-theoretic, inferential approach advocated, e.g., in Levinson (2000, 42–54).

[22] Van Lambalgen and Stenning emphasize this point in their Van Lambalgen and Stenning (2001, Section 9).

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