Ian Jarvie, Karl Milford and David Miller (eds.): Karl Popper - A Centenary Assessment. Volume III: Science. Ashgate Pub., Aldershot 2006, 105-112.

Section 1: Philosophy of the Physical Sciences

Karl Popper's Propensity Interpretation of Probability

Dr. Jacob Rosenthal
Universität Bonn
Philosophisches Seminar, LFB I
Am Hof 1
D-53113 Bonn, Germany
Email: Jacob.Rosenthal@uni-bonn.de

The propensity interpretation of probability was developed by Karl Popper in the 1950s as a successor to the frequency interpretation he had formerly adhered to.[1] According to the propensity interpretation, probabilities are properties of certain experimental arrangements, namely, tendencies of those arrangements to produce certain outcomes. More exactly: Imagine a random experiment with possible outcomes $A_1, A_2, \ldots, A_n$. That outcome A_i occurs with probability p_i means, according to the propensity interpretation, that the experimental set-up is endowed with a tendency or propensity of relative strength p_i to produce the outcome A_i. This is, of course, in need of explication, if 'propensity' or 'tendency' is supposed to be more than merely a new word for 'probability'. To give content to his conception, Popper offers us three main ideas: propensities are, first, dispositions of a certain kind, second, they are generalized physical forces or causes, and third, they are weighted possibilities. I will discuss these ideas in turn. The upshot will be that they all fail for the same reason: They presuppose, in one way or another, the concept of probability that should be interpreted by introducing the concept of propensity. They all lead to a conceptual circle.

First, propensities as dispositions. If a thing shows under certain typical circumstances a certain characteristic behaviour, we ascribe a corresponding disposition to the thing, as a persistent property of it. 'Something *x* is disposed at time *t* to give response *r* to stimulus *s* if and only if, if *x* were to undergo stimulus *s* at time *t*, *x* would give response *r*' (Lewis 1997, 143). A standard example for a disposition is solubility in water. Every lump of sugar would dissolve if given into water under normal circumstances, so every sugar lump has the permanent property of water solubility. The dissolution is the manifestation or display of this disposition. Dispositions are characterized through the corresponding displays. So, if propensities are dispositions of experimental arrangements, the question is: Dispositions to what? What are their characteristic manifestations?

[1] See the three articles Popper 1957, 1959 and 1967, the postscript to his *Logic of Scientific Discovery*, on which he worked in the 1950s and '60s, but which was not published before 1982, and finally the small book Popper 1990.

There are two answers to this question contained in Popper's works. According to the first, the manifestation of a propensity is a characteristic relative frequency in the long run (Popper 1957, 67; 1959, 35; 1967, 32–33). That a certain experimental arrangement has a propensity, or tendency, of relative strength p to produce the outcome A, means that, if the experiment were repeated very often (or even infinitely many times), the outcome A would occur in approximately (or even exactly) $100p\%$ of the cases. So propensities are dispositions of experimental arrangements to produce the possible outcomes with certain characteristic relative frequencies in the long run.

The problem with this answer is that it presupposes a nonprobabilistic connection between probabilities and relative frequencies, which definitely does not exist. Imagine a fair die, which, if thrown under normal circumstances, has equal probability for each number. If you throw the die repeatedly, each sequence of outcomes is possible, and in particular it is possible that each throw results in the same number. Since the different throws are independent, the second throw can of course have the same result as the first, the third the same result as the second, and so forth. This possibility is already implicitly contained in the characterization 'independent repetitions of the same (random) experiment'. So it is simply not true that a fair die would, upon repetition, lead to a series of outcomes in which each number occurs (exactly or approximately) with the relative frequency 1/6. That such a 'regular' series of outcomes occurs, is only highly probable, and it is the more probable the more often the experiment is repeated, but other series are nevertheless possible and remain so even in the (hypothetical) limiting case of an infinite number of repetitions.

To be a little bit more precise: If a fair die is thrown n times and the throws are independent, then every possible sequence of outcomes occurs with the same probability $(1/6)^n$. In particular, the series that consists only of sixes occurs with this probability. Its chance is no less than that of any other particular series of outcomes. With increasing n the proportion of such 'irregular' series **(**i.e., of series in which the six numbers are not approximately evenly distributed**)** to all series diminishes more and more, and so the probability of getting a regular series increases ('weak law of large numbers'). In the limiting case of an infinite number of throws the proportion of irregular outcome series to all series is zero, and so you get with probability 1 a series in which each number occurs with relative frequency 1/6 ('strong law of large numbers'). But this does not mean that you would get such a series for sure, because the irregular series are still there, and each of them has the same chance to occur as each of the regular series. Only the measure-theoretic proportion of the irregular series to all possible series is zero.

So, if a propensity is characterized as a disposition to produce certain frequencies in the long run, it is a disposition that is only probabilistically connected to its manifestations. The statement 'in the long run there would result such-and-such relative frequencies' is either false or to be understood as shorthand for 'in the long run there would *with high probability* result such-and-such frequencies'. But now a conceptual circle has emerged: Popper characterizes probabilities as propensities, and propensities as dispositions of experimental arrangements to produce certain relative frequencies in the long run. But since these occur only with high probability, the concept of probability that should be explicated via the idea of propensity is in fact presupposed by it. You cannot say what kind of disposition a propensity

is if you do not refer to probabilities. Of course there could be dispositions that are only probabilistically connected to their respective manifestations, and of course you are free to call such dispositions 'propensities' (see, e.g., Mumford 1998), but then you cannot interpret probability statements via reference to propensities.

The same problem emerges at once, and more obviously, if the manifestations of the propensities are not understood as certain relative frequencies in the long run, but rather as the outcomes of single experiments. This is the second answer contained in Popper's works as to what the manifestations of propensities are (Popper 1957, 67–68; 1959, 28, 37). The propensity of an experimental arrangement to produce the outcome *A* is displayed if and only if this outcome actually occurs upon carrying out the experiment. But since *A* occurs only with a certain probability *p*, the disposition of the experimental arrangement to produce *A* is, under the relevant circumstances, displayed only with a certain probability. Therefore the concept of probability is again presupposed.

So the result is that you cannot on the one hand give a propensity interpretation of the concept of probability, and on the other hand introduce propensities as certain dispositions. As dispositions are characterized through their manifestations, and probabilities are only probabilistically connected to observable events, each alleged candidate for a manifestation of a disposition called 'propensity' occurs, under the relevant circumstances, only with a certain probability. But it is this very concept of probability (in the context of physical theories) that should be interpreted by the idea of propensity, so it cannot be presupposed in an explanation of what propensities are.

Second, Popper compares propensities to physical forces (Popper 1957, 68–70; 1959, 27–28, 30–31, 37–38; 1967, 41–42; 1982, 93–95, 105; 1990, 12–14, 18–20). Doing this, he has more in mind than just that propensities are theoretical entities. He speaks of 'the idea of propensity as a kind of generalization of – or perhaps even an alternative to – the idea of force' (Popper 1982, 95; 1990, 12, 14), and he compares his introduction of propensities to the introduction of classical forces into physics by Newton (Popper 1957, 70; 1990, 13–14). The propensity interpretation of probability is in his opinion a new physical theory or hypothesis. But his further remarks on this point are too vague to be really helpful. Some suggest that he sees propensities as forces that vary probabilistically in strength or direction. That would indeed be a generalization of the idea of force, but there is no physical theory that uses this concept of randomly varying forces, and anyway, even if there were such a theory, the question of what probabilities in physics are would be as open as ever. The idea of probabilistically varying forces would be just another application of the concept of probability, and not an interpretation of it. So Popper's second idea, that propensities are something like generalized forces, is not very convincing either. One cannot say that Popper has proposed a new physical theory, or a modification of an existing one. What he does propose is a certain interpretation of the probabilities in physics or at least in quantum mechanics, an interpretation that makes these probabilities objective and applicable to the single case and that makes probability statements testable by means of relative frequencies. But it is one thing to aim at such an interpretation, and another thing really to develop it, and neither the concept of force nor the concept of disposition has proved helpful for this task.

Analogous remarks apply to the view of the propensity theory as a generalized theory of causality, i.e., to the characterization of propensities as a generalization of deterministic causes (Popper 1990). According to this view, propensities are causes or causal links of a certain strength. But what does it mean that an experimental arrangement is endowed with, say, a strong causal tendency to produce outcome *A* and with a weak tendency to produce outcome *B*? Does it mean that, upon carrying out the experiment, outcome *A* must occur? Does the stronger cause always beat the weaker one? This is, of course, not intended, for it is the situation of a typical random experiment we want to understand and model by the concept of propensity. So we have to say that the stronger cause is the one that succeeds with higher probability or the one that succeeds more often, and in either case we hit on the concept of probability. (Remember that, in a random experiment, everything that can be said about relative frequencies is qualified by probabilities.) Propensities, then, are causes or causal links that bring about the respective effects not necessarily, but only with a certain probability. Such a probabilistic theory of causality is evidently just another application of the concept of probability and not an interpretation of it.

Third, Popper says that propensities are weighted, or weights of, physical possibilities (Popper 1967, 32; 1990, 9–10). From this point of view the propensity theory appears to be an improvement of the classical conception of probability, according to which the probability of an event is the ratio of 'favourable' to 'equally possible' cases. One shortcoming of this idea is that in many examples there simply are no equally possible cases, e.g., when a loaded die is thrown. So you may generalize the classical conception by saying that the possible cases need not be equally possible, that some cases can be 'more possible' or 'easier to realize' than others, that, in general, 'weights' are attached to the possible cases and that the probability of an event is the sum of the weights of the favourable cases. And, if you like, you can call these weights, or possibilities thus weighted, 'propensities'. But this can hardly be called an interpretation of probability. 'Probability' is simply replaced by new words with the same meaning. Remember here a second well-known shortcoming of the classical conception, namely that 'equally possible' means exactly the same as 'equally probable', so that the definition of probability as the ratio of favourable to equally possible cases involves a conceptual circle. This criticism applies equally to Popper's propensity theory, if it is presented in this way. That a certain possible outcome has 'greater weight' than another is just to say that it occurs with greater probability. If you try to avoid this conceptual circle by referring to frequencies instead, i.e., by saying that 'greater weight' means 'occurs more often', you are again confronted with the problem that relative frequencies are only probabilistically connected to probabilities, or propensities, or weighted physical possibilities, or whatever you want to put in this place, so that you are, after all, still caught in the conceptual circle.

I conclude that Popper has not succeeded in providing a suitable interpretation of probability in the context of physics. 'Propensity' or 'tendency' are no more than pictorial names for objective single-case probabilities or objective chances. If one thinks about probabilities in nature, one may have the idea that there are 'tendencies' in the world that pull in different

directions with different strengths. No wonder that the idea of force also comes to mind in this context. But if you want to leave the level of mere associations and develop an interpretation of probability based on these ideas, you immediately become involved in a conceptual circle. For what can it mean that a certain possible outcome of a random experiment has a greater tendency or propensity to become actual than another possible outcome? It means either that the first outcome occurs with higher probability, in which case the circle is most obvious. Or it means that the outcome would, upon repetition, occur more frequently, which is either false or to be taken as shorthand for 'would with high probability occur more frequently', in which case the conceptual circle emerges again. Popper's various attempts to explicate his talk of 'propensities' or 'tendencies' of experimental arrangements all lead back to this dilemma.

Does this mean that the propensity interpretation of probability has to be dismissed? Not necessarily. First, one could say that according to the propensity interpretation, 'probability' is just a fundamental, irreducible concept that cannot be explicated any further by reference to other concepts. One would then drop the talk about dispositions, forces, weighted physical possibilities and so on and simply say that propensities are real, physical entities, objective and applicable to the single case, and ascribable to experimental arrangements by means of relative frequencies – period. The problem with this answer is that it leaves the crucial properties of propensities unexplained: Why do propensities obey the probability calculus? Why are they connected to observable relative frequencies, and how do they explain the occurrence of those frequencies? There is no suitable bridge principle that connects propensities and relative frequencies, as there is in other cases where a theoretical entity is connected to an observable one, because any such connection would have to be qualified by higher-order probabilities that must be interpreted in turn.

Second, following Popper, other writers have developed varieties of the propensity interpretation that are perhaps more promising. Among these, I want to mention Mellor (1971) and Lewis (1980). Both of them introduce propensities, or objective chances, not via their connection with relative frequencies, but via their connection with subjective probabilities, or rational degrees of belief. Popper would certainly not have approved of that, because from their point of view the subjective concept of probability, i.e. the concept of probability as rational credence, is fundamental. But it seems clear, first, that there are such things as subjective degrees of belief, measurable through betting quotients, and that coherent betting quotients obey the calculus of probability. Second, it seems plausible that this or a similar concept of probability must be fundamental, because 'probable', 'probability' and related notions are first of all epistemic notions that have their origin in our limited knowledge in general, and that the idea that there are probabilities in nature comes only later. Third, it is clear that if there are such things as objective chances, i.e., probabilities in nature, they certainly constrain rational credence, or more precisely: the objective chance of an event provides the appropriate degree for the belief in its occurrence. Otherwise the term 'objective probability' would simply be out of place. Whatever it is in nature that is given the name 'objective probability', it certainly would not deserve this name if it did not constrain rational credence. So it seems to be a promising idea to found a concept of objective chance on a

theory of subjective probability plus a bridge principle of the indicated kind that connects chance and credence.

This program was carried out concisely by David Lewis (1980, 1986, 1994). Its problem is that according to it we know nothing about objective chances or propensities except that they are entities that constrain rational credence. Everything else follows from that. To model a chance phenomenon according to the propensity interpretation means to assume that the world contains entities that provide appropriate degrees of belief. How they manage to do that remains obscure. Chances or propensities have a certain normative power that is not further explained, and we know nothing about them except that they have this power. Or, to put it the other way round: chance is nothing but objectified credence, subjective probability inscribed into nature. This is no doubt very strange, but attempts to do better by connecting propensities to certain observable entities have failed so far. Of course there might be other observable entities than relative frequencies to which propensities are connected. What immediately comes to mind here are physical symmetries, symmetries in nature. But first, there is no indication that suitable symmetries, from which the objective chances can be inferred, always exist, and second, there may well be several different symmetries that lead to different probabilities (Bertrand's paradoxes). So physical symmetries are no promising candidate either to provide us with a general and nonprobabilistic connection between objective chances (propensities) and observable entities. But without such a connection objective chances (propensities) remain obscure entities.[2]

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[2] Thanks to two anonymous referees for valuable comments and criticisms.

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